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The development of pre-service teachers' competence to teach mental calculation strategies

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Background: There is a concern in South Africa that pre-service teachers do not have the required knowledge to teach mathematics in primary school and to develop learners' number sense. In this study, pre-service teachers taught the mental strategy of bridging through ten through a structured teaching sequence from the Mental Starters Assessment Project (MSAP) materials as a work-integrated learning opportunity.

Aim: We ask the question: How do the MSAP materials support pre-service teachers in competently teaching mental mathematics?

Setting: Thirty-eight Bachelor of Education (Foundation Phase) third-year preservice teachers from an Eastern Cape university participated in this study.

Methods: Participants taught the strategy during their Teaching Practice, quantitatively analysed the results of their classes and reflected on the experience in a questionnaire and focus group interviews.

Results: Results indicate that the teachers were relatively successful in their teaching of the strategy; however, all indicated that they taught the sequence for a more extended period than recommended. Qualitative responses provide evidence of the teachers' development in their knowledge of learners and their characteristics, general pedagogical knowledge, pedagogical content knowledge and knowledge of educational contexts.

Conclusion: This study offers evidence of the professional learning of pre-service teachers that resulted from taking an integrated approach to facilitating a mathematics teaching methodology course through requiring a work-integrated learning component.

Contribution: We argue that such an approach is necessary for pre-service teachers to be adequately prepared for the challenges of teaching mathematics in the South African classroom.

Keywords: pre-service teachers; teacher education; mental strategies; foundation phase; mental starters assessment project.

Introduction

Current research into teaching and learning mathematics in the South African primary school classroom shows that Foundation Phase learners rely primarily on unit counting to calculate (Schollar 2008). As learners progress into the Intermediate Phase, they tend to be taught formal procedures and rules for calculating (Graven et al. 2013). Standard algorithms typically require knowledge of basic facts and procedures, while mental calculations are more complex as they focus on the structure of numbers and operations and their relationships as well as on basic facts (Rathgeb-Schnierer & Green 2019). When learners are required to calculate mentally, they are given the opportunity to develop a wide repertoire of strategies for calculating and, as such, develop flexible ways of thinking. However, with the well-documented persistence of unit counting into the Intermediate Phase and beyond, it is clear that learners are not developing such strategies for calculating.

The Curriculum and Assessment Policy Statement for mathematics (South Africa. Department of Basic Education [S.A. DBE 2011]) in the Foundation Phase proposes that teachers spend the first 10 min of every maths lesson developing learners' mental mathematics. The distinction between mental mathematics and written forms of mathematics is that mental mathematics promotes calculation with numbers (e.g., 24 + 25 = 25 + 25 – 1) as opposed to digits (e.g., 24 + 25, 4 + 5 = 9 and 2 + 2 = 4) (Westaway & Vale 2021). Mental calculations enable learners to draw on a variety

Note: Special Collection: Mental mathematics and number sense in the early grades.

of strategies to ascertain that are most efficient in each context (Pourdavood, McCarthy & McCafferty 2020). Pre-service teachers need to be able to identify and use the most efficient strategy for calculations and need to learn how to similarly develop this with their learners.

There is a concern in South Africa that pre-service teachers do not have the required content and pedagogical content knowledge to teach mathematics in primary school and to develop learners' number sense (Courtney-Clarke & Wessels 2014). Many researchers (e.g., Adler et al. 2009) suggest that teacher education institutions are complicit in the poor performance of learners in South Africa as pre-service teachers are not prepared to teach mathematics in such a way to develop flexibility, efficiency and accuracy when calculating. More recent research has brought to light some worrying evidence that there are low levels of content knowledge development from initial registration for a Bachelor of Education degree to graduation (see Bowie, Venkat & Askew 2019). Pre-service teachers need to be able to identify and use the most efficient strategy for calculations and need to learn how to similarly develop this with their learners. As Ebby (2000) argued, teacher education should also support pre-service teachers to develop mathematics teaching approaches that might need to depart from what they experienced in school. This is certainly the case for many South African pre-service teachers who may have learned mathematics at school from teachers who may not have had adequate content knowledge and pedagogical content knowledge themselves (see Fleisch 2008; Shulman 1987). Nilsson (2008) explained that it is through learning processes grounded in classroom practice that pedagogical content knowledge can be developed. Nilsson (2008) further argued that:

> [*T*]eacher educators need to provide opportunities for student-teachers to examine and integrate new knowledge and beliefs about teaching and learning into their existing knowledge and beliefs [*through*] activities such as observing, analysing and reflecting upon one's own … teaching. (p. 1285)

This research project aimed to provide such an opportunity to the pre-service teacher participants.

This research aimed to develop pre-service teachers' content knowledge of a variety of mental calculation strategies and to develop their ability to competently teach these strategies. In the first phase of this research, the participant teachers were introduced to a selection of mental strategies through the Mental Starters Assessment Project (MSAP) materials (see Graven et al. 2020) over a period of 15 months (February 2022–May 2023). This was done with the aim of assisting the participants to develop their own flexibility in using mental strategies. The focus of this article is on the second phase, where the participants each taught the strategy of bridging through ten to a Foundation Phase class over a 3-week period in July and August 2023. When applying the strategy of bridging through ten, a learner would calculate the sum of 25 and 9 by adding 5 to 25 to get to 30 and then would add the remaining 4 to 30. The research question we ask in this article is: '*How do the mental starters, as a structured teaching sequence, support pre-service teachers in competently teaching mental mathematics?*'

In this article, we respond to the main research question mentioned above through attending to the following sub-questions:

1. To what extent are pre-service teachers able to teach the bridging through ten mental starters effectively?
2. What is the nature of pre-service teacher learning when teaching mental mathematics using the mental starters?

The Mental Starters Assessment Project

The MSAP materials were developed by colleagues from the South African Numeracy Chair, WITS Maths Connect, the SA.DBE and a number of international researchers (Graven et al. 2020). In the teaching sequence, learners write a 5-min pre-test prior to engaging in eight 10-min structured lessons that aim to develop a mental strategy, e.g., bridging through ten. Following these lessons, the learners write a 5-min post-test to assess whether they were able to make progress. The mental starters aim to improve learners' rapid recall of number facts and their strategic calculating and strategic thinking aligned with the mental strategy in focus. Strategic calculating is defined for this project as 'developing facility with using the focal strategic approach for efficient calculating-by-structuring' (Venkat, Askew & Graven 2023:672), and strategic thinking is defined as 'developing awareness of the underlying structure of the strategic calculation approach in each unit in open calculation representations' (2023:672). These materials are written for Grade 3 (ages 9–10 years).[1]

As explained by Graven and Venkat (2021), a theoretical premise on which this work has been developed is Askew's (2012) argument that when looking at mathematical working, fluency, reasoning and problem-solving are the most visible of Kilpatrick Swafford and Findell (2001) strands of mathematical proficiency. Askew (2012) argued that learners need to be fluent in effective and efficient strategies in order to allow working memory capacity for more complex learning and problem-solving. The MSAP materials thus focus on the development of six mental strategies: bridging through ten, jump strategies, doubling and halving, rounding and adjusting, re-ordering and linking addition and subtraction. In these materials, the terms 'rapid recall', 'strategic calculating' and 'strategic thinking' are used to refer to Askew's (2012) fluency, reasoning and problem-solving. Rapid recall skills are the 'basic skills or known facts that need a level of automaticity for efficient use of a calculation strategy' (Graven & Venkat 2021:28).

The beginning of each MSAP lesson starter comprises short warm-up activities that aim to improve learners' fluencies in these basic skills and known facts. Thereafter, the strategy is introduced to learners, and they are offered an opportunity to practice the strategy in focus with the aim of developing their ability to use the strategy to inform their thinking and

1.See https://www.education.gov.za/MSAP2022.aspx.

calculations. The sequencing of the presentation of the calculations themselves is also carefully structured to draw learners' attention to the structure of the number relationships and the relationships between the operations (as drawn from the work of Watson & Mason 2005).

Venkat and Roberts (2022) identified that one priority for effecting improvements in early-grade mathematics teaching and learning is for investment in and support for the design and implementation of well-researched teacher development programmes. With the concern about quality mathematics education in South Africa extending to concerns about pre-service teachers' own proficiencies and their level of preparedness to teach mathematics effectively by graduation, we have been introducing them to the MSAP materials in this project. Venkat and Roberts (2022:217) mentioned, in particular, that 'carefully designed and well-structured learners' workbooks need to be deliberately used as a vehicle for professional development'. The MSAP materials are such a resource with a well-structured teacher handbook with detailed instructions on how to teach the mental strategies and carefully sequenced learner work prescribed.

This has happened at two levels. At the first level, and in the first phase of this research, the pre-service teacher participants were taught the strategies as presented in the MSAP materials. They wrote the pre-tests, engaged with the eight lesson starters and then wrote the post-tests. At the second level, the PSTs were engaged with understanding how the MSAP materials are structured and how to teach the MSAP strategies and then engaged in teaching the bridging through ten strategy to a Foundation Phase class.

Theoretical framing

The theoretical framing of this article is drawn from the work of Shulman (1987), who specifies seven categories of the knowledge base of teachers. These categories were used to code the responses from participants to the questionnaire. The seven categories, according to Shulman (1987:8), include:

1. content knowledge;
2. general pedagogical knowledge, with special reference to those broad principles and strategies of classroom management and organisation that appear to transcend subject matter;
3. curriculum knowledge, with a particular grasp of the materials and programmes that serve as 'tools of the trade' for teachers;
4. pedagogical content knowledge, that special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding;
5. knowledge of learners and their characteristics;
6. knowledge of educational contexts, ranging from the workings of the group or classroom, the governance and financing of school districts, to the character of communities and cultures; and
7. knowledge of educational ends, purposes and values and their philosophical and historical grounds.

We would argue that all of these categories of knowledge are important in building a knowledge base for teaching mathematics. Our interest, in this research, is to learn of which categories there is evidence of the pre-service teachers having developed in their awareness and knowledge through the teaching of the mental starters. We use Shulman's (1987) categories to code the responses provided by the pre-service teachers in their reflections on their teaching experiences.

Research methods and design

Participation in the research reported on in this article required the pre-service teachers to teach the mental strategy of bridging through ten, according to the MSAP structured teaching sequence, in their Foundation Phase classes during their teaching practice in July and August 2023. Participants ranged in age from 21 years to 30 years and the classes that were taught ranged in size from 20 to 45 learners.

Process of participation

Following their 15 months of engagement with the Mental Starters (from February 2022 to May 2023) to develop their own knowledge and skills in using mental strategies, the Bachelor of Education (Foundation Phase) third-year pre-service teachers engaged in three 2-h workshops in May 2023 in which they worked to adapt the materials for bridging through ten to be suitable for teaching across the Foundation Phase. This involved the close examination of the curriculum documents to establish the expected level at which Grade 1 and Grade 2 learners would be working in Term 3 of the academic year and close work with the MSAP tests, worksheets and teaching sequence. The adaptation was focused on restricting the number range used in the tests, worksheets and teaching sequence to be within that which Grade 1 and Grade 2 learners should be fluent in working in. No adaptations were made to the representations, including part-whole diagrams and number lines, used in the teaching sequence or the materials. No adaptation was necessary for Grade 3. This was done with the support of the two authors and then standardised by the two authors prior to their use in the classrooms.

The participants were engaged in 5 weeks of teaching practice in July–August 2023, during which each pre-service teacher was teaching daily in a Grade 1, 2 or 3 class. The bridging through ten strategy was taught in the final 3 weeks. This teaching practice experience was the first for these pre-service teachers in which they were required to take full responsibility for teaching the classes. Data collection included a questionnaire and a series of 40-min focus group interviews, all administered in September 2023. Thirty-eight participants completed the questionnaire, and 34 participants participated in focus group interviews conducted by the two authors. Each focus group interview involved the participation of pre-service teachers who taught the same grade, not exceeding six per interview. The questionnaire required them to quantitatively analyse the pre- and post-test results of their learners by reporting on changes in results for the rapid recall items and the strategic thinking and

calculating items. It also required participants to reflect on whether the mental starters had been of benefit to their learners, what adjustments they made to the teaching sequence as they taught and what they learned from the experience. The focus group required the participants to provide more elaboration on their questionnaire responses and as such was guided by the same questions as appeared in the questionnaire. In order to analyse the qualitative responses to the questionnaire and interviews, the participants' responses were coded according to the seven categories of Shulman's knowledge bases for teaching.

Data analysis

The data that were analysed included the quantitative data supplied by each pre-service teacher reflecting the results of the pre- and post-tests that were conducted with each class. This allows us to comment in response to research sub-question 1, which asks to what extent the pre-service teachers were able to achieve success in their teaching of the bridging through ten strategy. The Foundation Phase learners' work does not form part of the data for this study, but the self-report data from the pre-service teachers about the relative success or otherwise of their whole classes are part of the data. In addition, the participant responses to the questionnaire and the focus group interview were analysed with Shulman's (1987) categories as the guiding framework. Each response was categorised according to whether the comment was made in relation to content knowledge, general pedagogical knowledge, curriculum knowledge, pedagogical content knowledge, knowledge of learners, knowledge of educational contexts or knowledge of educational ends, purposes and values. The specific questions were the following:

1. Were there any questions in particular that your learners struggled with on the tests? Please describe which questions the learners struggled with and the type of answers they were giving to them.
2. Please comment on whether you feel the mental starters were of benefit to your learners and explain the reasons you feel they were either of benefit or not to your learners.
3. Please explain any adjustments that you might have made as you taught the mental starters. You may have made adjustments to the tests or worksheets or the teaching sequence. Describe any changes you made and provide a reason for making those changes.
4. Did you feel the teaching sequence was clear enough for you to know how to teach the mental starters? Please explain your answer.
5. Were the tests and worksheets suitable for your learners? Please give details to explain your answer.
6. What did you learn as a beginner teacher from your experience of teaching the mental starters?

To further support the data from the questionnaires, pre-service teachers were asked to discuss their experiences and what they had learned in a focus group interview. The interviews were guided by the same questions as those included in the questionnaire. They were transcribed using otter.ai, and the researchers reviewed and corrected the automatically generated transcript. Excerpts from the transcripts were used to further support the findings from the coded questionnaire responses.

The first author organised the responses from the completed questionnaires into a spreadsheet while noting initial ideas pointing to possible coding according to Shulman's (1987) categories. Thereafter, a random selection of half of the responses for each question was coded separately by the authors, and agreement was reached about the coding. Agreement on coding was above 80%, and where there was disagreement, this was resolved by discussion. The first author then continued to code the complete data set, and this was reviewed by the second author to establish full agreement on coding. A count was made of the frequency of each code in relation to the number of responses to which the code applied.

Ethical considerations

An application for full ethical approval was made to the Rhodes University Education Faculty Research Ethics Committee, and ethical approval was granted on 17 March 2023. The ethics approval number is 2023-5441-7475. Renewal of this ethics approval was granted by the same committee on 24 January 2024, and the ethics approval number for the renewal is 2024-5441-8299. Gatekeeper permission to conduct research with students was granted by the registrar on 17 March 2023. The pre-service teachers were required as part of their Foundation Phase Studies 2: Mathematics course to engage in this teaching, but participation in the research was voluntary. Those choosing to participate in the research gave written informed consent to participate in completing a questionnaire and in a focus group interview in September 2023. The questionnaire and interviews were conducted in accordance with the ethical standards of Rhodes University, the South African National Health Research Ethics Council and the 1964 Helsinki Declaration and its later amendments.

Presentation and discussion of findings

In order to respond to the research question about whether the mental starters support pre-service teachers in competently teaching mental mathematics, we discuss data in relation to the three sub-questions. To understand: (1) to what extent pre-service teachers are able to teach the bridging through ten mental starters effectively, we share quantitative evidence of the results of the classes who were engaged in this learning with the pre-service teachers. To explore: (2) the nature of pre-service teacher learning when teaching mental mathematics using the mental starters, we use Shulman's (1987) categories of the knowledge base of teachers to analyse the qualitative data from the questionnaires and focus group interviews in order to categorise where there is evidence of learning for the pre-service teachers and what the nature of that learning was.

Quantitative evidence of the effectiveness of the teaching

The focus of this study is on the pre-service teachers' experiences of using the bridging through ten mental starters and their learning, and therefore, our focus is not on the learners' performance. However, it is relevant to note the extent to which the pre-service teachers were able to effectively teach the strategy if we are to understand their experiences. We therefore report on the mean results of the classes as analysed by the pre-service teachers. The pre-service teachers conducted the pre- and post-tests provided in the MSAP materials with their learners and were guided by the researchers to construct a summary of their results for the purposes of this research. The participants were asked to report on certain quantitative measures, including the pre-test class averages; the number of learners improving, dropping and remaining at the same level and the post-test class averages. They were supported in calculating these measures by the researchers.

It is relevant to report on the extent to which the pre-service teachers were able to successfully implement the mental starters in order to make any judgements about the nature of their learning. As one such measure, Table 1 provides a summary of the overall changes in mean scores across the classes in each grade, as reported by the participants.

According to the data provided, the pre-service teachers appear to have been able to achieve success when viewed over all classes. Three classes dropped in the rapid recall section of the test between pre- and post-testing, and six classes dropped in the strategic thinking and calculating section of the tests. Interestingly, there was no class group that dropped in both sections of the test.

It is worth noting that the results achieved by the pre-service teachers are comparable to those achieved by qualified teachers in the Eastern Cape and Gauteng during early trials of the bridging through ten strategy (Graven & Venkat 2021). For rapid recall, the Eastern Cape Grade 3s improved by 14% points and the Gauteng Grade 3s improved by 19% points. The pre-service teachers' Grade 3 classes showed a more modest percentage-point improvement (10.9%-pts); however, the pre-service teachers' classes, on average, achieved higher post-test scores for rapid recall (68.8%) when compared to the Eastern Cape Grade 3s (40.7%) and the Gauteng township school (61.3%). It is only learners in the more-resourced suburban Gauteng context that outperformed the average performance of the pre-service teachers' Grade 3 classes. In addition, with respect to the Eastern Cape Grade 3s in Graven and Venkat's (2021) study, the 15.23%-point improvement achieved by the pre-service teachers is higher than that achieved by the Eastern Cape qualified teachers. The mean percentage achieved for strategic thinking is also substantially higher.

It is possible to compare the Grade 3 pre-service teachers' performance relative to the scores achieved by qualified teachers in Graven and Venkat's (2021) study. This is not possible for the Grade 1 and Grade 2 classes, as this is the first time that the materials have been adapted for use with these grades. The results in Table 1 for Grades 1 and 2 would, however, indicate that the pre-service teachers experienced similar success in those settings. The 18.24%-point improvement on strategic thinking for the Grade 1 classes is the highest improvement of any of the groups and would suggest that this strategy may be suitable for Grade 1s to learn.

Despite these indications of success, it is important to note a limitation: all pre-service teachers interviewed indicated that they did not use the MSAP sequence without making adaptations. The adaptation that all who were interviewed indicated making was to extend the amount of time spent on the strategy. All indicated that once they started teaching, they realised that the aims were unachievable in eight 10-min lessons, and thus, they adapted the materials to teach for more extended periods. These extended periods ranged from extending the 10 min to up to 30 min for some or to slowing down the progression to take place over more than eight lessons. Many also indicated that their classes were unable to engage in the pre-tests without some teaching intervention, which perhaps explains the higher pre-test scores. The more extended teaching time would also explain the higher post-test scores. It can, however, be argued that the pre-service teachers experienced some success in their teaching of the mental strategy of bridging through ten, and we learn in the following sections some of the factors that might have contributed to this.

The nature of pre-service teacher learning

There were 243 analysable responses from the questionnaire. The percentage of responses that were coded in each category is provided in Table 2.

Excerpts are attributed to students by mentioning the grade taught, and their assigned letter code, e.g. Grade 3L refers to the pre-service teacher in a Grade 3 classroom assigned the letter code 'L'. In addition, the question the excerpt is in response to is provided (e.g. Q6 refers to question 6). If the

TABLE 1: Changes in mean percentages across all classes per grade.

Grades	Rapid recall (%)			Strategic calculating and Strategic thinking (%)			Total (%)		
	Pre-Test	Post-Test	%-pt change	Pre-Test	Post-Test	%-pt change	Pre-Test	Post-Test	%-pt change
Grade 1 (8 classes)	42.5	54.8	12.32	17.0	35.3	18.24	34.0	45.0	11.03
Grade 2 (14 classes)	54.3	66.7	12.37	46.1	52.5	6.37	51.6	62.0	10.43
Grade 3 (13 classes)	57.9	68.8	10.90	35.9	51.2	15.23	50.5	63.7	13.20

TABLE 2: Shulman's knowledge categories.

Shulman's (1987) categories	Questionnaire responses (%)
Content knowledge	2.0
General pedagogical knowledge	24.0
Curriculum knowledge	6.5
Pedagogical content knowledge	13.0
Knowledge of learners	42.0
Knowledge of educational contexts	12.0
Knowledge of educational ends, purposes and values	0.5

excerpt is from a focus group transcript, the focus group the excerpt is taken from is provided (e.g. Grade 3, FG1 is the first focus group of Grade 3 pre-service teachers).

Below are some excerpts to show exemplars of the coding of questionnaire responses. The first excerpt is coded as containing content that can be considered evidence of *pedagogical content knowledge*, *knowledge of learners and their characteristics* and *knowledge of educational contexts*:

> 'I used count on more as it was one of the strategies they understood more because most of them were confused by the bridging through ten strategy. On the question which says 8 less than 10, I explained it to them as 10 minus 8. [*pedagogical content knowledge]* I also made adjustment on time especially on the post-test because on the pre-test they were disadvantaged by time. Most of learners in my class were slowly when writing so I had to adjust time to get fair results, because most them know these questions but they are slowly in working them out. [*knowledge of learners and their characteristics*] I also had to translate all questions into isiXhosa, I was explaining the questions in isiXhosa because they do not understand English [*knowledge of educational contexts*].' (Grade 1E, Q3)

The following excerpt provides another example of a pre-service teacher displaying *pedagogical content knowledge*:

> 'I broke down the mental starters. I taught the part part whole separately on the mat, the equals to sign on the mat and the number line on the mat in different days. I did not teach solving problems using a number line because learners were still confused with the number line. I also changed the worksheet. I used the grade 1 worksheet first to see if they are understanding the method that I'm teaching or not. I also changed the wording and the number range. So instead of saying 8 less than 10 I would say 10 -8 to make learners understand what the question is saying better.' (Grade 2N, Q3)

The following four excerpts show examples where we coded for *general pedagogical knowledge content knowledge curriculum knowledge* and *knowledge of educational ends, purposes and values*:

General pedagogical knowledge:

> 'I have learned that pre-tests are for you to realise what the learners understand and their abilities then post-test is more like how the lesson aims were achieved.' (Grade 2G, Q6)

Content knowledge:

> '[*N*]ot really, I did not feel ready to teach mental starters at all, for me I still struggle with it so I did not feel that I have acquired enough to share with someone.' (Grade 1F, Q4)

Curriculum knowledge:

> 'Yes, the mental starters lesson outlines built on one another – ensuring that each mental starter touched on what the learners learned the previous day.' (Grade 1B, Q4)

Knowledge of educational ends, purposes and values:

> 'Every child has the potential to learn when given the chance.' (Grade 3K, Q6)

We see in the quote in relation to general pedagogical knowledge the Grade 2 pre-service teacher indicating having learned about the general role of pre- and post-assessments of knowledge, while in the quote reflecting aspects of the Grade 1 pre-service teacher's content knowledge, there is a comment on their confidence in their ability to make use of the mental strategy themselves. The illustrative quote presented in relation to curriculum knowledge shows a realisation of how the mental strategy is revealed through the structuring of the mental starter lessons, while the final quote reflecting on the knowledge of educational ends, purposes and values is a strong statement of the Grade 3 pre-service teacher's sense of the final outcome for the learners of the teaching progression.

There is evidence to support that the participants reflected particularly in relation to four of the sub-domains of Shulman's (1987) categories: knowledge of learners, general pedagogical knowledge, pedagogical content knowledge and knowledge of educational contexts. In particular, their knowledge of learners and their characteristics as mathematics learners are reflected in their responses. Some illustrative quotes of evidence that pre-service teachers were learning about this are provided below:

> 'I mean, also, very good. Recall the bonds of ten very well. But then when it comes to writing it's as if they're just confused. It's as if they can't read the sums without you reading it out loud together.' (Grade 1, FG1)

> 'We did ability groups, the top group was very quick at getting it and getting how it works, breaking down the numbers using the number line using the beads, and ask them to imagine it as a number line. But the middle, the middle group, and the lower group got easily frustrated when I asked them to imagine it as a number line.' (Grade 2, FG1)

> 'They won't use the number lines because you can actually see that some of them they don't use the number lines, they will get back to counting on their fingers instead of using the number line.' (Grade 3, FG1)

> 'At first, majority of the learners used the count from/count on strategy to add and subtract 2-digit numbers. The learners are now able to add and subtract 2-digit numbers using the bridging through 10 strategy.' (Grade 3L, Q2)

> 'To an extent yes, some learners grasped it, but others did not at all. Some of them just guessed numbers and the others were so busy counting to get an answer that they did not even write anything down at all.' (Grade 3E, Q5)

> 'The tests and worksheets were not suitable for my learners as most of them were struggling to understand the questions especially those that were starting with written instructions in order to do the sum.' (Grade 2E, Q5)

> 'I've learnt that it is important to start from the beginning with learners and to gradually teach a content over and over again and not think that learners remember what was taught the previous day. I've learnt to be more patient and understand that some work can be too difficult for others while easy for some as there were learners that would want us to move on to the next questions while the other learners are still struggling.' (Grade 1G, Q6)

It is evident from these quotes that the pre-service teachers have recognised the abilities and needs of the Foundation Phase mathematics learners they were teaching. For example, as seen in the quote above, the Grade 3L pre-service teacher noted the strategies the learners were already able to use. They have noticed what they are and are not able to do and then proceeded to give accounts for how they have enacted their developing general pedagogical knowledge and mathematical pedagogical content knowledge. One example of this would be the Grade 1G pre-service teacher's realisation in the quote above that what was taught the previous day needed to be revisited before presenting new work that required that knowledge.

Some examples of displays of their developing general pedagogical content knowledge are provided below. In each of these quotes, we read of general pedagogical strategies that the pre-service teachers had tried and learned more about, for example, making use of repetition, working with ability groups, giving clear instructions, breaking down strategies into smaller steps and working to make learning enjoyable for the learners:

> 'I learned that teaching a new strategy takes a lot of patience and time. The learners understood something the one day and couldn't remember what to do the next day. It takes constant repetition.' (Grade 1B, Q6)

> 'The first 2 days I took the whole class for the mental starters. Thereafter I decided to take the learners in their ability groups. Working with a smaller number of learners at a time, proved to be more efficient as learners felt more confident to share and make mistakes. I could also easily see any confusion and address it in the moment.' (Grade 1A, Q3)

> 'I learned that not every child will understand, which means that I as a teacher I will need to be patient with learners as it might take time for them to understand and make sure that I give clear instructions to learners.' (Grade 2J, Q6)

> 'I learnt that teaching a strategy from scratch is not easy but the more you do it using ability groups your experience will be better. I also learned that breaking down a strategy to get to where you want to be is very important as learners need to have a foundation before doing the strategy.' (Grade 2N, Q6)

> 'I have also learned that I need to be confident in my teachings and a fun lesson really makes learners engage and participate.' (Grade 3G, Q6)

In addition to providing evidence of developing general pedagogical knowledge, some participants provided responses that could be argued to show some development of mathematical pedagogical content knowledge. Some examples are listed below and include mention of becoming able to derive their own additional examples and integrating correction of errors into the teaching sequence:

> 'I was able to know what to do and to adjust the explanation to fit Grade ones. It equipped me to be able to come up with my own examples when I see that learners were struggling to grasp content.' (Grade 1E, Q4)

> 'When I was teaching the lesson on bridging through 10, I started off by reflecting on the pre-test. I identified the sums most of the learners struggled with and did corrections on them, one of which was the part unknown sum. I had to integrate it and make it part of the bridging through 10 lesson.' (Grade 3L, Q3)

> 'I did use some of the context to make examples for them on the mat after the pre-test so they can have a better understanding especially the questions were what's on the left side must be equal to what's on the right e.g. 23+9=23+3+__ to help them understand because that questions was the ones they were struggling with.' (Grade 3F, Q3)

While the examples displaying developing mathematical pedagogical content knowledge are fewer than for general pedagogical knowledge (see Table 2), it is important that there are these examples among the responses. Some of the pre-service teachers were able to connect what they were seeing the learners do in the class with ways in which to structure their teaching to address a teaching goal informed by the learners' performance. We argue that it is this type of activity, where pre-service teachers are tasked with teaching a structured sequence (and therefore have an expected learning path in mind), that heightens their sensitivity to noticing deviations from what they expected. This is supported by Nilsson's (2008) argument that pedagogical content knowledge is acquired through learning processes grounded in classroom practice. These pre-service teachers were then able to try their own adaptations, including inserting additional examples and review of test items into the teaching sequence and measure their relative success in their chosen teaching approach through assessment of the learners (see Table 1) and reflection on the process as seen in the quotations provided herein.

The final category of the four in which the pre-service teachers showed evidence of development is in their knowledge of educational contexts. As Venkat and Roberts (2022) indicated, investment in and support for the design and implementation of well-researched teacher development programmes is a priority for effecting improvements in early grade mathematics teaching and learning. We would argue that understanding the experiences of novice teachers in implementing the mental starters within their context is an important part of the research required to inform this programme. It is evident in the examples presented below that the reality in the contexts that the pre-service teachers were teaching meant that many learners were not necessarily ready to learn the strategy. Much of this information came to light in the focus group discussion, where they elaborated on their experiences across different contexts. We share some extracts below:

> 'They were not benefiting the learners because most of them could not understand the question and there were lots of explanation that I needed to do before they could write the mental starters.' (Grade 3D, Q2)

> 'So, you see now it's no longer beneficial because it gave me a grade three worksheet for them to read and they're not ready … imagine wanting to teach someone a strategy, but they can't deal with numbers on it.' (Grade 3, FG2)

Many pre-service teachers indicated that learners were not ready for the mental starters and cited reasons including that they did not recognise all the digits up to 10 (Grade 1) or had not worked with two-digit numbers before (Grades 2 and 3) or did not know how to work with number lines (all Grades). However, it was interesting to hear in the focus groups that the participants recognised that teaching the mental starters, and specifically in this case bridging through ten, was important and could be attained with some better planning. The response below from a participant in the Grade 3 focus group clarifies:

> 'I think that it [*the mental starters*] must be implemented from grade one, from earlier. Even easier questions in grade R. Yeah, because then it's not something new introduced only in Grade 3. I mean, read through them, it's very overwhelming because I feel the teachers haven't taught them like this. Yeah, it's important to actually do some of the mental maths with the learners. Like my class, they can't subtract at all.' (Grade 3, FG1)

Another member of the group echoed this idea and mentioned that he thought that the mental starters should be started at the beginning of the year in developing the pre-requisite skills required to learn the strategy and become routine in the classroom and that the component skills and representations be introduced more gradually. He ended by stating that, with reference to the context in which he was teaching:

> 'I think this experience was good and beneficial for [*the writers*] to understand that it is a little bit unrealistic.' (Grade 3, FG2)

This is in light of the majority of the pre-service teachers indicating that they were not able to adhere to the time allocation if they were to teach the strategy effectively in the contexts in which they were teaching:

> 'The adjustments that I've made is that of time allocation and explaining to the learners the questions. The mental starters took more time than that required in the mental starters as there were learners that didn't understand the concept being taught therefore I had to teach it to the learners for 25 min – 30 min a day.' (Grade 1G, Q3)

> 'The time was too short for them, therefore, I had to give them more time to finish the starters and explain some of the problem they were struggling in.' (Grade 2K, Q3)

The ability to recognise the time constraint as limiting learning and making the requisite adjustments to allow for proper learning, we argue in itself evidence of the pre-service teachers' developing professional knowledge in this project.

Conclusion

While the study is limited by its reliance on self-reported learning, rather than a researcher-led assessment of knowledge, it is clear that according to the participants, there was rich professional learning for the pre-service teachers who taught the bridging through ten strategy. The structured teaching sequence gave the participants a resource to support their teaching of the strategy, but the limitations of this teaching sequence in the classroom quickly became evident to them when they were met with learners for whom the pre-requisite skills were not in place as expected. This required them to make adjustments, and this challenge enriched what they were able to learn about the content and about children and teaching mathematics.

In relation to the MSAP's structured teaching sequence for bridging through ten, we learn that for pre-service teachers, the structured guideline for teaching provided an entry point from which they were able to initiate their teaching. However, this was only to the extent that it provided a guiding sequence within which they were able to adjust for the prior knowledge and capabilities of the learners they were working with. The participants expressed a desire for the designers of the mental starters to consider adapting some for earlier grades and to provide guidance to teachers on how to build the pre-requisite skills required to learn the strategies. All recognised the importance of the mental starters, but their feedback about the realities of the classroom environment is important to take forward into discussions about the rollout of the materials and how to support teachers with teaching the strategies.

What was achieved in this research project was to embed a teaching component into the lecture-based subject on mathematics teaching methodology in the Foundation Phase. The structured mental starters had been a topic that was explored at length in the 15 months prior to this work-integrated learning term where pre-service teachers were placed in Foundation Phase classrooms to trial the teaching sequences. The requirement to enact what they had learned in lectures, and then reflect on the experience of doing so, would seem to have resulted in rich professional learning. Evidence has been presented of pre-service teachers reporting knowledge of learners and their characteristics and developing in their skills of noticing the mathematical thinking of learners. There has also been evidence presented of their developing general pedagogical knowledge, as well as some evidence of developing mathematical pedagogical content knowledge. This shift from learning exclusively the university-based lecture room to undertaking independent (but supported) teaching resulted in the participating pre-service teachers deepening their professional knowledge and developing in their general and mathematical pedagogical content knowledge. We argue that such an approach is necessary for pre-service teachers to be adequately prepared for the challenges of teaching mathematics in the South African classroom.

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Authors' contributions

P.V. and L.W. were involved in the conceptualisation, design, implementation, analysis of data and writing up of this research.

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Data availability

The data that support the findings of this study are available from the corresponding author, P.V., upon reasonable request.

Disclaimer

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