Review: Van den Heuvel-Panhuizen M, Kuehne C & Lombard A-P. 2014. *Learning Pathway for Number in the Early Primary Grades*. South Africa.

Mike Askew, Monash University, Melbourne, Australia / Visiting Professor, School of Education, University of the Witwatersrand.

I cringe at the fashion of reducing everything to a set of discrete points – 'the five keys to eternal happiness', 'twenty-seven ways to beat procrastination' – yet find myself attracted to the 4 E's model of cognition: that cognition is embodied, embedded, enactive, and extended. Far yet from being a coherent model in the sense of general agreement over what each of these terms means, and despite sounding reductionist, the model seems to have traction through bringing together differing theoretical positions and suggesting that cognition cannot be accounted for by just one of them.

For example, Lakoff & Núñez (2000) argue for embodied origins of mathematics, but the power of mathematics also resides in the way that it extends our 'natural' understandings. And while the Vygotskian position of cognition arising from the move from the interpersonal – the embedded and enactive – to the intrapersonal is popular, it seems to lack explanatory power about how it is that what originates between people comes to be located within the individual. The proprioceptor emphasis of embodied theories (that cognition originates through our literal, common senses of position and the movement of our bodies in space) has, I think, potential to unlock this connection between the social and individual.

Thus the 4 E's together look greater than the sum of the parts and, as Menary (2010:462) points out:

> "The once homogenous framework of cognitivism is being replaced by a multidimensional analysis of cognition as incorporating our brains, bodies and environments."

Thinking of cognition as arising from the 4 E's also provides a unifying framework that accounts for developments in mathematics education originally arising from different

South African Journal of Childhood Education | 2014 4(2): 213-216 | ISSN: 2223-7674 |© UJ

theoretical traditions. For example, the highly influential 'Children's mathematics' (Carpenter, Fennema, Franke, Levi & Empson 1999), with its detailed analysis of the problem types that lay the foundation of arithmetic and the informal solution strategies that learners bring to such problems, might now be subtitled 'Embodied and enactively guided instruction', rather than 'cognitively guided instruction'.

Similarly, Freudenthal's Realistic Mathematical Education (RME) (Freudenthal 1975), with a similar emphasis on enactively solving problems from which mathematics can be 'realised', might be seen as anticipating a 4 E's view of cognition.

So it is welcoming that Marja van den Heuvel-Panhuizen, Cathy Kuehne and Ana-Paula Lombard's *Learning Pathway for Number in the Early Primary Grades* (hereafter referred to as LPN) brings together the Dutch research and pedagogies, and education in South Africa and explores how this might play out in foundation phase (Grades R to 3) mathematics teaching and learning in a fashion that could be seen as recognising a 4 E's view of cognition.

The approach to teaching foundation phase number is, according to LPN, grounded in five principles:

1. The context principle: problems set in meaningful contexts have the potential to provoke mathematical activity.
2. The level principle: teaching needs to support learners in moving from context-bound strategies to more general ones.
3. The activity principle: mathematics arises from human activity (as opposed to being a collection of Platonic objects pre-existing to human activity).
4. The interaction principle: the importance of recognising the social nature of learning.
5. The guidance principle: learners have to be guided to 'reinvent' mathematics.

(Adapted from Heuvel-Panhuzen, Kuhne & Lombard, pp. 6-7)

Principles 1 and 3 can be read as embracing an embodied and enactive approach; Principle 2 as demonstrating how knowledge is extended; and Principles 4 and 5 as recognising the embedded nature of cognition through interactions with others.

However, having been set out in the introduction, these principles then remain rather implicit in the body of the book, leaving the reader to figure out how they inform the advice subsequently given in LPN.

Take, for example, the coordination of the context principle and the level principle, and the advice that LPN provides on 'Stage 2' subtraction:

> "When solving subtraction problems such as 7 – 3 (either presented as a context problem or as a pure number problem), similar strategies can be applied as in addition problems."
>
> (p. 62)

The work of Carpenter and colleagues (Carpenter, Fennema, Franke, Levi & Empson 1999) has shown that the type of context problem presented can dramatically affect the solution strategies that learners employ. 'Seven girls were in the room and three left. How many girls stayed in the room?' and 'Seven girls were in the room with three boys. How many more girls were there than boys?' can both be modelled with 7 – 3, but are likely to provoke learners to use different strategies, including, in the latter case, counting up from three to seven.

The discussion of strategies for 7 – 3 in LPN emphasises counting back strategies, and although the point is made that, for 9 – 7, "instead of taking away 7 from 9, a learner can add on from 7 and find that that difference is 2" (p. 63), there are no examples of specific context problems that might lead to these different strategies. Neither is the question of when a learner might interpret a 'pure number problem' as difference, rather than 'take-away', explicitly addressed. My experience of working with teachers, in South Africa and elsewhere, suggests that choosing appropriate context problems from which the mathematics can emerge is challenging. Without examples and discussion of the principles behind the choice of examples, teachers may simply stay with problem structures they are most familiar with, for example subtraction primarily as 'take-away' problems. It is a pity that LPN does not more explicitly point out how the five principles have influenced the advice provided and give more examples of the practical implications of such principles, particularly in terms of the sorts of context problems that can support the emergence, or reinvention, of mathematics.

Rather than these five principles providing some framing structure for the book, LPN is organised around four 'stages of number development': emergent number concept, counting-and-calculating, calculating, and advanced calculating (p. 9). The origin of these stages is not clear. Labelling them as 'development stages' suggests that they might be grounded in research on learning, but no indication of whether that is the case is provided. (In fact, no references are provided in the book at all. Whilst I appreciate that the intended audience is the practicing teacher, is it a pity that the interested reader is not provided with any idea of where to look if they want to find out more about the theoretical or empirical underpinnings of LPN).

LPN's four-stage model suggests development along a model of (1) numbers to 5 or 10 (and beyond); (2) operations to 10 and numbers to 20 (and beyond); (3) operations to 20 and 100 (and beyond); (4) operations to 1000 and numbers to 10 000 (and beyond). Such a model, based on logical increases in the numbers, does not seem to reflect current thinking about learners' growth in knowledge, which we know is much less linked to specific number ranges and based more on understanding number structure and properties (see, for example, Cowan, Donlan, Shepherd, Cole-Fletcher, Saxton & Hurry 2011). The authors of LPN do note that by adding 'and beyond' in each level (the range of numbers is not intended to be a 'fixed demarcation'), but this still sets up normative expectations of learning as quite linear.

In fact, these 'stages of development' seem to fit more with the CAPS requirements, as the number ranges match those set out in the CAPS summary provided with LPN (pp. 4-5). If that is the case, then better I think to explicitly acknowledge this than leave the reader to wonder about the status of these 'stages'.

LPN does provide a good overview of the use of artefacts such as bead strings, images and number lines to support the development of counting and calculating strategies, and teachers who are not sure about the power of these to support learning will find much of value here. But there is scant attention to working with fractions and little use of measurement contexts, so the emphasis is firmly on whole number calculation. Any reader seeking pathways for 'number' in the fullest sense of the word will need to supplement the book with other resources.

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