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Teachers' perceptions about collaboration as a strategy to address key concepts in mathematics

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Background: The aim of the study is to investigate teachers' perceptions about peer collaborative work in designing lessons as a team helped them to identify threshold concepts in the teaching and learning of foundation phase mathematics in Motheo District of Education.

Methods: A qualitative approach, with a case study design, was used to combine data from observation and focus group discussions, interviews and group task sheets.

Classroom observation was conducted during a workshop conducted by a subject advisor from the Motheo District of Education in collaboration with the researcher. Teachers were purposively selected from seven schools in the Motheo District of Education based on cluster sampling as a way of reviving their professional development through acquisition of mathematical teaching skills involving innovative approaches to teaching and learning of early childhood mathematics. Seven mathematics teachers, one from each school, were interviewed during the workshop.

Results: Underpinned by a collaborative theory, the findings of the study revealed that peer collaboration in mathematics teaching was key to helping them (participant teachers) identify threshold concepts in mathematics that they had initially found difficult as individual teachers. This assisted them in teaching the subject effectively at the foundation phase level. The study, furthermore, established that collaboration by mathematics teachers was necessary in order to overcome the paucity of global mathematics teaching skills for early childhood mathematics, to foster learners' knowledge of mathematical concepts and to stimulate their interest in the subject.

Conclusion: It is recommended that more structured collaborative work amongst teachers in general should be encouraged to enable teachers overcome the problem of content gap in their area of specialisation.

Keywords: collaboration; key concepts; mathematics teaching and learning; teacher perceptions; learners; Motheo Education district; South Africa.

Introduction

The dawn of democratic governance in South Africa in 1994 has been followed by a series of reforms in the education system (Khuzwayo 2005), with much attention being focused on teacher preparation and readiness for classroom teaching and learning. The Department of Education's policy documents are based on the assumption that teachers' content knowledge has a significant influence on learners' learning. Research conducted in many parts of the world reveals that teachers' content knowledge makes a difference in their classroom instructional practices as well as in their learners' achievement (Mishra & Koehler 2006; Newborn 2001; Shulman 1986). Research conducted by Loewenberg Ball, Thames and Phelps (2008) states that teachers must really know the subject that they teach, because if they themselves do not know the subject well, they will find it difficult to have the knowledge they need to help learners learn the content. Robert-Hull, Jansen and Cooper (2015) advocate that the way and manner in which candidates are prepared to be teachers in many parts of the world have a critical influence on what teachers can do to change the teaching environment and what their learners learn in school. Teachers should be prepared adequately for their profession.

Shepherd (2012) alluded to the fact that for quantity and quality, South African school teachers were ranked low, based on learner performance because of poor foundations in mathematics and science compared to other developing countries in Africa, as most developing countries are still finding their feet in the development of Early Childhood Education (ECE)/foundation phase (FP) programme as public service for public good, and their efforts are characterised by a lack of commitment to the implementation of various policies by their respective governments

(Britto, Yoshikawa & Boller 2011). He further indicated that many mathematics and science teachers could not teach the subjects properly because of the lack of content knowledge for FP, and as a result, poor teaching contributed to the poor performance amongst learners in mathematics and the sciences (Shepherd 2012). Teaching of mathematics is a fundamental process that should demonstrate both subject content knowledge (SCK) and the pedagogical content knowledge (PCK) (Shulman 1986). Being equipped with such knowledge enables teachers to be more effective, flexible, fluent thinkers and confident in their use and application of knowledge to the FP learners and processes. However, Rigelman (2007) explained that mathematics representation and explanations given to learners in the classroom are often characterised by the teachers' poor conceptual understanding or knowledge of the subject.

Newborn (2001) contends that in America, the types of knowledge essential for the teaching and learning of mathematics in elementary schools have been a research area for the last 40 years. Borasi (1990) maintains that mathematics, like any other subject at school is an important means of bringing a basic understanding of logic and invention to people. Therefore, it would appear that mathematical knowledge, like any other subject knowledge, is valued and must be presented in a manner that portrays conceptual understanding. This statement is affirmed in research conducted by Tsang and Rowland (2005) who state that to teach mathematics effectively, teachers must have good mastery of the substantive and syntactic structures of mathematics. Teachers must not only be capable of telling learners about the accepted facts, concepts and principles of different branches of mathematics, but also be able to explain to learners why a particular mathematical principle is deemed warranted, why it is worth knowing and how it relates to other principles within the same branch and across other branches of mathematics (Tsang & Rowland 2005). This is in line with a statement by the Ghazali (2019) that:

> [*S*]tudents need to learn mathematics in ways that enable them to recognize when mathematics might help to interpret information or solve practical problems, apply their knowledge appropriately in contexts where they will have to use mathematical reasoning processes, choose mathematics that makes sense in the circumstances, make assumptions, resolve ambiguity and judge what is reasonable in the context.

However, it seems that in many countries, teacher preparation programmes for FP learners for the rural areas sometimes delay for no apparent reasons and do not facilitate the acquisition and development of the necessary content knowledge required by teachers to teach the curriculum to perfection. Some of these programmes are not implemented early at preschool for the learners to acquire skills but put off until they attained the age of 6 years or more, depending on the child's ability to manipulate objects (Preston & Haines 2014), hence the current study.

The current study was triggered by a request by a senior education specialist in the Motheo education district of the Free State province of South Africa to organise an intervention programme to support mathematics teachers in the district with the ultimate aim of improving the quality of teaching at FP level and learners' performance in mathematics in the early stage of FP level. Prior to this intervention programme, the researcher realised that there was a deficiency in teachers' professional learning and development, through collaboration in the district, as a way of increasing interest to support the progressively complex skills learners need from teachers to learn mathematics in preparation for further education and work in the 21st century. Darling-Hammond, Hyler and Gardner (2017:v) state that sophisticated forms of teaching are needed to develop student competencies such as deep mastery of challenging content, critical thinking, complex problem-solving, effective communication and collaboration and self-direction. According to these researchers, this is a very effective professional development (PD) strategy needed to help teachers learn and refine the pedagogies required to teach mathematics skills. However, research has shown that many PD initiatives appear ineffective in the educational system in many parts of the world in supporting changes in teacher practices and student learning, hence the current study. Further interrogation by the researcher revealed teachers' views, which indicated that they experienced problems in handling some of the mathematical concepts and that they had to consult their colleagues in most cases for assistance. This lack of SCK, it seems, was the result of forced redeployment where redeployed teachers were required to teach mathematics. A workshop was organised by the researcher and mathematics subject advisors in order to address teachers' lack of SCK. This paper, reporting on the workshop, aims to identify teachers' perceptions about peer collaboration amongst mathematics teachers as a way to address difficult key concepts or to identify the threshold concepts in early childhood mathematics teaching and learning. To address the above problem, the following main question is posed:

> How can collaborative work on designing lessons as a team help teachers identify threshold concepts in the teaching and learning of early childhood mathematics?

Literature review: Peer collaboration amongst teachers

Peer collaboration and co-operation amongst teachers are regarded as key factors in improving teachers' PD. International researchers such as Ni Shuilleabhain and Seery (2018), as well as Fullan and Hargreaves (1992), argue that in order for any relevant and successful fundamental change to occur in the classroom, teachers must be encouraged at all times to collaborate with their peers. This serves as part of a learning curve for the teachers in their communities in order to overcome the problem of content gap experienced in some topics within some subjects. Teacher collaboration in a community can provide a powerful structure within which

individual teachers can attempt to understand and reflect on new approaches to teaching and learning relevant to their own school context, learners and culture (Dogan, Pringle & Mesa 2016; Vescio, Ross & Adams 2008). What works best for learners is what teachers, as well as the Department of Education, agitate for in schools.

Studies have shown that the concept of teamwork is pervasive within the United States Army but found to be limited in the world of academia (Charbonneau et al. 2010). A review of related literature in international contexts has revealed that effective collaboration amongst peer teachers for lesson planning is a form of development of teacher knowledge. Furthermore, collaboration encourages a more learner-centred approach to teaching and learning of mathematics (Dudley 2013; Lewis, Perry & Hurd 2009; Murata et al. 2012; Ni Shuilleabhain 2016). In collaborative engagement with peers, with the view of helping each other, teachers are able to deal directly with the curriculum, identify the aims and objectives of their teaching and plan lessons which are definitively linked to both the philosophy and content of the curriculum (Cajkler et al. 2014; Takahashi & McDougal 2016). It is believed that the feeling of wanting to help one another to accomplish a goal is like leaders empowering their subordinates to accomplish a certain mission or different missions in the classroom environment. This is relevant to everyday teachers' interactions with learners. Peer collaborative work aims at planning and designing lessons through reflection on action and helps teachers to understand mathematical concepts (either difficult or easy), stimulate critical thinking amongst learners and promote learning through hands-on activities. The ability to persistently and carefully consider what and how teachers teach, and to reflect on their actions as teachers to determine what works best for their learners, is central to successful teaching because reflection is a vital component of learning how to teach well (Myers 2012). In some cases, some teachers and learners do not easily understand mathematical concepts. In those instances, collaboration and co-operation amongst teachers is the only way for teachers to gain this understanding.

In this study, teachers' perceptions about collaboration were introduced as a new school-based teacher model of PD facilitated through a workshop. Teachers of a particular community met and collectively discussed and identified key mathematical concepts that needed to be taught but with which they were not familiar. Doing so facilitated and improved the teaching of mathematics and further assisted teachers to improve their understanding of certain mathematical concepts.

Theoretical framework

This study is underpinned by Vygotsky's sociocultural theory which states that because learning takes place amongst individuals, it is an inherently social process activated through the zone of proximal development (Dillenbourg 1999). The study also incorporates Shulman's five proposed content-specific domains of teacher knowledge, as reported by Pasley (2011). Shulman (1986:9) defines SCK as 'the amount and organisation of knowledge in the minds of a teacher' and includes knowledge of mathematics facts, concepts and procedures and their relationships. Pedagogical content knowledge is described as a particular form of content knowledge that represents how the aspects of content are to be taught for conceptual understanding and mathematics knowledge. In teaching mathematical concepts, teachers' knowledge of the concept should consider the importance of the shaping effect of learners' experiences. It is through experiences that the impact of human culture on understanding and acceptance occurs, and it is where an individual constructs the rules and conventions of language with the extensive functional outcomes manifested around us in human society where learning occurs, as advocated by Jaworsk (1994). Vygotsky's views on this theory contributed significantly to social constructivist epistemology, which dwells on how learning is mediated by collaboration in accordance with the context and by sharing of personal experiences with peers through open discussion in the process of overcoming teaching and learning obstacles. This type of learning is a type of social interaction which concentrates on cognitive development of individuals through discussion and sharing of information (Lantolf & Thorne 2006; Lin 2015).

The aim of every teacher is to establish a professional learning community conducive for his or her learners as one of the effective means for enhancing teachers' PD effort in the teaching environment. Studies have shown that different methods or forms such as theory-driven approach and Manabu Sato's learning community theory can be applied to structure all components of teacher PD workshops that impact positively on teachers' teaching beliefs, knowledge and skills acquisition for better teaching and learning (Darling-Hammond et al. 2017; Lin & Wu 2016). Accordingly, this can be done in different forms based on the specific needs of individual schools and teachers' needs or objectives. The need for professional support through collaboration would help teachers establish a platform convenient for collegial dialogue amongst colleagues in order to pool wisdom and ideas that support their understanding of difficult areas in order to optimise learning and teaching. Teachers embark on this collaborative discussion to ensure that their students understand the need to learn mathematics in ways that enable them to recognise when mathematics might help to interpret information or solve practical problems, apply their knowledge appropriately in contexts where they will have to use mathematical reasoning processes, choose mathematics that makes sense in the circumstances, make assumptions, resolve ambiguity and judge what is reasonable in the context (Commonwealth of Australia 2008:11). This was an important aspect in this study as participant teachers collaboratively shared their ideas in order to identify strategies of teaching certain concepts that they considered as mathematics content gaps. The purpose

of the workshop was to create a learner-centred approach at the FP level combined with collaboration to develop greater understanding of certain mathematical concepts for effective teaching and learning of mathematics especially at the FP level (Lewis 2016).

Methodology

This research followed a qualitative approach with a case study, as the research sought to understand the experiences of teachers who collaborated as a way to identify and understand certain mathematical concepts that were perceived by the participating teachers to be difficult to teach at the FP level as well as the relevant principles that enhance effective teaching and learning of ECE. Following a discussion with the mathematics district curriculum specialist (DCS), mathematics teachers from certain schools especially where we have FP classes were invited to participate in the research. Participants (30 mathematics teachers) were purposefully selected from various schools in a Free State Education District by means of a cluster sampling technique. At least one teacher was selected from each cluster, with a total of 15 clusters based on FP levels. The aim was to have at least one representative from each cluster, whereby he or she would share the skills and strategies acquired through this collaboration with his or her cluster members at a convenient time in teaching of ECE learners. The purposive sampling technique was used to select 30 FP level mathematics teachers for the purpose of this study. Ten mathematics teachers each from the following three categories were used for the selection of the participant teachers: 10 mathematics teachers from high-achieving schools either in the current or previous teaching experience (five from urban schools and five from rural schools), 10 mathematics teachers from average-performing schools (five from urban schools and five from rural schools) and 10 mathematics teachers from low-performing schools (five from urban schools and five from rural schools) in the Free State province. These schools were identified by relying on the Annual National Assessment results for the subsequent 3 years in the province and have experience of teaching FP levels, skills and strategies used in teaching these children for meaningful understanding, as most of their parents do not have time for their children at home. The purpose of the selection strategy was to share a variety of opinions from different teaching and learning environments.

Observation, focus group discussions, group task sheets and in-depth interviews were used as data collection strategies. Evidence was gathered by observing a group of 30 mathematics teachers who participated in a workshop (see Figure 1, Figure 2 and Figure 3), with seven participants being interviewed. Because the researcher did not want to interfere unnecessarily in the lesson planning and discussion of concepts, the data collected were essentially limited to feedback on the topics selected by the teachers for discussion, based on their objective for the discussion and curriculum coverage. The dataset gathered for this study over the progression of the sequence included:

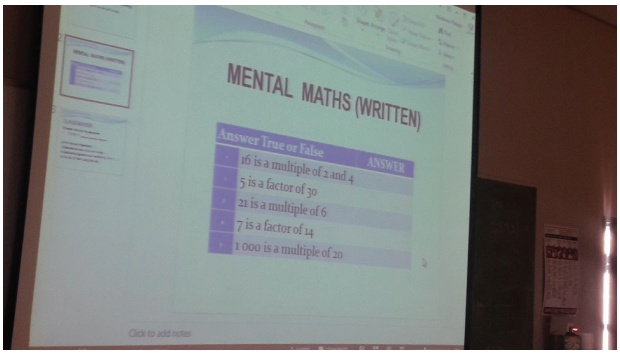

FIGURE 1: Activity 1 on mental maths.

FIGURE 2: Peer collaboration during the workshop.

FIGURE 3: Peer collaboration during the workshop.

- Objectives regarding the collaboration to identify threshold concept effective for ECE.
- Action plans to solve problems under discussion using physical or real objects of ECE.
- Reports and feedback pertaining to the resolution for the problem (P1) elaborated on by individual teachers in the study.
- Final reports on the resolutions of subsequent problems to be discussed or solved with learners in the class regarding the actual teaching of learners.
- Contingency plans that show teachers' reflective practices used to identify other threshold concepts in early childhood-level mathematic teaching and learning and to assist learners further to solve problems where possible.
- Reports from individual teachers indicating the success or failure of a collaboration in identifying certain mathematical concepts.

- The way forward for identifying more threshold concepts for future discussion.

In addition to the data listed above, the researcher requested information from the participant teachers on the performance of their learners in their various schools prior to the collaboration, the mathematics curriculum and textbooks used in teaching and learning. The researcher also used his field notes report collected to assist in triangulating the data gathered from the teachers. Teachers were later interviewed for further details to augment their outputs.

Instrument

Observation and focus group discussions, interviews and group task sheets were used for data collection (see Figure 1, Figure 2 and Figure 3), which, according to Maree (2016), can be used to collect from the sources.

Research procedure

Before any discussion of threshold concepts took place, three of the participant teachers taught three different lessons, with the group observing. An observation schedule, designed according to a strand or strands of teaching mathematics for proficiency, was used to identify particular concepts and how these were presented to the group. The lessons were also videotaped for the purpose of reflection and discussion. Doing so assisted the researcher in critically analysing teachers' mathematics knowledge.

The video recording assisted the group in successfully identifying the listed threshold concepts for group discussions. During the course of the discussions, equal opportunities were given to each participant teacher to contribute, share their opinions and clarify points in relation to the identification of threshold concepts that assist learners with problem-solving. This was followed by discussion and in-depth interviews. Throughout the study, participants were recorded and analysed who, when teaching and during in-depth interviews, actually demonstrated interesting mathematics knowledge for identifying key concepts in their presentations and in the task sheet.

Trustworthiness and credibility of the study

Based on the purpose of this study, relevant participants and suitable instruments were selected for this study. The data collected in this research were recorded electronically and transcribed for analysis. The participant teachers were given the opportunity to review the transcriptions to ensure that they were accurate. The researcher used semi-structured questions to guide the interviews, video recorded the participants as they presented their views based on the questions asked or under discussion. The researcher also studied the lesson plans for lessons presented during this study. The researcher also ensured descriptive validity which actually goes into the details of what actually had been gathered at the field; hence he used open and transparent procedures in gathering the raw data without any fabrication of any part of information (Maree 2016).

Data analysis

Prior to the analysis, all recordings were transcribed. The researcher analysed the data collected, that is, raw data captured from the responses of the participants who were asked the same set of questions, including some body language as well as facial expressions exhibited by the participants, which were recorded in field notes. An inductive analysis approach was used, and the data analysis was guided by specific evaluation objectives, which involved a detailed reading of the raw data to derive the concept, themes or models. This understanding of inductive analysis in research starts with an area of study and allows the theory to emerge from the data (Miles & Huberman 1994). An analysis of transcriptions, in terms of answering questions and teaching using appropriate mathematics knowledge added significantly to the richness of the research data.

Ethical considerations

Ethical clearance was obtained from the University of the Free State, Ethical Clearance Number: UFS-HSD2018/0395. 17/07/2018

Findings and results

The purpose of the research was to establish mathematics teachers' perceptions about peer collaboration amongst teachers as a way to address difficult key concepts or identify the threshold concepts in early childhood mathematics teaching and learning. The data analysed in this research were gathered through observation, focus group discussion, interviews and task sheets. After teachers engaged in collaborative discussion and shared ideas with one another and with the researcher (see Figure 1, Figure 2 and Figure 3), the participant teachers were able to create and construct appropriate learning opportunities that assisted them in identifying threshold concepts in mathematics for teaching and learning of primary school mathematics. The results are presented according to focus groups followed by observations, task sheet responses and, finally, the interview data.

Focus group and discussion-based results of professional development

The teachers formed groups comprising at least seven teachers per group of four (see Figure 2 and Figure 3). Gender disparities were considered to avoid bias and cluster sampling avoided aspects relating to age, experience, race and resources of the various school. Questions were randomly distributed to group members for discussion according to the themes that had emerged from observation during lesson presentation. The researcher and research assistants carefully monitored what was discussed when questions were presented to a particular group, and all

information was recorded and checked by the assistants. Before focus group discussions commenced, every participant teacher was made aware that when a question was directed at a particular group to elicit comments or viewpoints, they could answer or contribute if no further contributions were forthcoming from that group as a way to develop participant teachers professionally (see Figure 3). The purpose was to ensure that there was positive change in the lives of teachers regarding their teaching and learning of ECE as well as their learners. Darling-Hammond et al. (2017:v) define effective PD as structured professional learning that results in changes in teacher practices and improvements in student learning outcomes. According to them, the use of relevant methodology in structuring effective PD through collaboration involves seven features which include being content focused, incorporating active learning, supporting collaboration, using models of effective practice, providing coaching and expert support, offering feedback and reflection and being of sustained duration, which provides teachers with adequate time to learn, practise, implement and reflect upon new strategies that facilitate changes to their teaching practice. This idea was adhered to in this study; therefore, during the focus group discussions, all group members were asked to pay attention to the answers given by the particular group in order to deliberate successfully and achieve better understanding of the emerging categories or themes from the discussion. During the focus group discussions, the following comments were provided by participants in their various groups about the value of collaboration in workshops (see Figure 3):

> 'Actually, in our group, we were really impressed by the way various teachers solved problems on fractions. For the fact that we are mathematics teacher does not mean that we know everything. Some of the skills, methods, strategies demonstrated by some teachers through this research has been overwhelming in dealing with ECE mathematics. We are really blessed to have collaborative work like this that exposed us to different opinions of solving mathematics problems or different ways of identifying mathematical concepts which makes teaching of mathematics in FPs very ease.' (Group 3, teacher 2)

> 'We need to embark on this kind of project very often whether we like it or not because we get to know many things during group discussion which is really difficult for most of us to understand when we plan alone as individual teachers in our respective schools. We could now see different ways of addressing mathematical problems or identifying mathematical concepts that will definitely help us to guide our future learners by showing different skills, activities, concepts and models to make our teaching enjoyable and understandable to our learners which we never knew at the beginning (See Figure 1). Really, we thank the organisers of this programme because we have known now that working together as groups and planning and sharing of ideas openly like this is really valuable and helps to build our skills of identifying some concepts with ease in ECE.' (Group 1, teacher 3)

> 'Unlike the teacher who only challenged the learner by mere talking without any illustrations is not a good way or procedure of teaching FP learners. The reason why most teachers cannot develop their skills and strategies by applying practical work in their teaching is that, in most cases, the problem may come from the Department of Education, whereby you are being forced to complete the syllabi at all cost without taking into consideration the cognitive level of the learners. When this happens, you will be forced to teach abstractly without doing illustrations of this nature and this does not help you as a teacher teaching FP learners. This must in fact, be looked into by those at the management level in order to recruit teachers with relevant skills and strategies to improve teaching and learning of FP mathematics.' (Group 1, teacher 1)

> 'There no doubt that anybody here will oppose collaborative work looking at what we have acquired here through challenging, discussion, probing and demonstration. We need to advise the department to make provision for this type of workshop which we hardly get so that we will be able to share our ideas. We are lucky that DCS for mathematics is here with us and we hope to see him taking this request to the provincial manager.' (Group 4)

Teachers' perceptions on observation and task sheet analysis

During classroom observation of lessons presented by individual teachers, the focus was on the way teachers presented their lessons in relation to their content knowledge and PCK so as to identify key concepts based on the topic presented in line with what Darling-Hammond et al. (2017) advocated in order to develop deep mastery of challenging content, critical thinking, complex problem-solving, effective communication and collaboration and self-direction. It was observed that even though teachers were able to mention some of the key concepts in the activity projected (See Figure 1), some teachers found it difficult to identify certain key concepts in some of the topics they have to teach. For example, based on Figure 1, an activity was projected on the screen and teachers were asked to identify the key concepts in this mental maths activity.

This is what some teachers had to say:

> 'This topic is about 'factors' and 'multiples of 2, 4 and 20.' (Teacher C)

> 'This particular example is very easy but at times, you find it very difficult to identify some key concepts in topics like fractions, 3D shapes and naming of intercepts in geometry, so we always need to do collaborative work like this to empower us to overcome any barriers in teaching of early childhood mathematics of this nature.' (Teacher A)

> 'Even though collaborative work is good for teachers for PD; however, it needs time and commitment to make it work effectively. But looking at our time schedules nowadays, it's not easy to have teachers come together to have this kind of engagement. I can't imagine having a collaborative work like this in my lifetime.' (Teacher C)

Teachers' perceptions on group discussion for professional development

During group discussions, comments and questions raised by the teachers on the task sheet provided opportunities to understand the content and produce golden rules for effective teaching and identification of mathematical concepts. This form of providing coaching and expert

support teachers, which involved the sharing of expertise about content and evidence-based practices, focused directly on teachers' individual needs as well as their learners for PD, as indicated by Darling-Hammond et al. (2017).

Peer collaboration amongst mathematics teachers is a way to address difficult key concepts or to identify the threshold concepts in early childhood mathematics teaching and learning, which threaten teachers' feelings and confidence. The solutions presented for questions revealed the kind of mathematical knowledge teachers possessed and how that knowledge paved the way for them to identify certain concepts in some aspects of their teaching and learning of mathematics regarding both content and method. Teachers shared ideas through group discussion, and it revealed what was happening in their classes to promote the development of mathematical proficiency. By identifying mathematical concepts during teaching and learning, examining how teachers present their lessons to their learners and linking ideas in different contexts produce meaningful learning. How the concepts could be linked to real-life situations were checked in line with the kind of concept being taught. The categories and themes identified based on knowledge of mathematics (content) and knowledge of instructional practices (method) were used to compare what Kilpatrick et al. (2001) call the 'five strands of mathematics proficiency', namely, conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition. This was followed by fellow teachers' judgment on their presentations by the other participating teachers.

A fraction question was posed to the teachers so that they could demonstrate their skill in identifying mathematical concepts to facilitate the understanding of their learners.

Question 1: If the answer to a sum of a particular problem is $\frac{2}{7}$, demonstrate how you would assist a Grade 7 learner in order for him or her to write down the relevant numbers correctly. What will the numbers be? Explain your answers.

The following were the responses by the groups:

> **Group A:** $\frac{1}{4}+\frac{2}{4}=\frac{3}{4}$ $\frac{1}{7}+\frac{1}{7}=\frac{2}{7}$ i.e., one-seventh of a banana plus one-seventh of another banana gives two-sevenths.
>
> **Group B:** $\frac{1}{4}+\frac{1}{28}\rightarrow$ a quarter of an orange added to one-twenty-eighth of an orange gives two-sevenths of an orange.
>
> **Group C:** $\frac{1}{7}$ and $\frac{1}{7}$ or any fractions equivalent to the stated ones. Add numerators when denominators are the same.
>
> **Teacher D:** $\frac{1}{4}+\frac{1}{28}$ we are not sure how they got their answer to be $\frac{2}{7}$. There could be a mistake somewhere. We want them to explain their answer as stated.
>
> **Group B leader:** As Group C already explained, any fractions equivalent to the stated ones. Add numerators when denominators are the same, i.e., the LCM of 4 and 28 is 28, which gives an answer of $\frac{8}{28}=\frac{2}{7}$, so the answer is correct. Alternatively, both denominators could be written the same by looking at the LCM of both denominators which gives $\frac{28}{112}+\frac{4}{112}=\frac{32}{112}=\frac{2}{7}$. The answer is $\frac{3}{4}$. It means fractional values are added; sum is the result of addition.
>
> **Group A:** The numbers could be $\frac{1}{7}+\frac{1}{7}$. This is because addition of these fractions will result in the answer $\frac{2}{7}$.

During focus group discussions, an attempt was made by the various group members to answer this question; however, some only provided single solutions without explaining their chosen answer. Group A, for instance, simply wrote $\frac{1}{7}+\frac{1}{7}$ without giving further explanation, thus failing to consider alternatives that could have been used. Considering the purpose of the study, as well as teachers' experience of teaching mathematics at the Senior Phase level, this question might have been too simple and they might have believed further explanation was unnecessary and not worth giving. What is interesting though is Group D's lack of response to or opinion on the question but their challenge to the credibility of Group B's answer, even though the first three groups gave different answers. What should be questioned, though, is that all these teachers provided single solutions. This probe by Group D gave a platform for the teachers to deliberate on ways that helped them to understand and identify certain concepts easily, which they had initially found difficult.

In actual fact, establishing that the sum of the numbers is $\frac{2}{7}$, some teachers went ahead during discussion and provided the answer $\frac{2}{7}+\frac{1}{7}-\frac{1}{7}$, as used by some individual teachers in the various groups as an explanation to justify their solutions of $\frac{1}{4}+\frac{1}{25}$.

Initially, the teachers believed the sum involved two numbers being added; however, as discussions continued, it was evident that the participant teachers knew the sum was not just a mere adding of only two numbers, but addition of different numbers. It was clear that teachers found engaging in such a collaborative discussion was useful because skills for identifying mathematical concepts could be acquired easily. However, time and commitment are needed to make this method or approach work effectively.

Discussion of results

Extensive research into teacher communities is not common in mathematics education; therefore the need for collaboration is worth considering (Gellert 2008). This study was conducted through a teacher collaboration workshop comprising (30) mathematics teachers from various schools in a Free State Education District. The strategy used was peer collaboration teamwork that helped to identify key concepts in mathematics teaching and learning. The purpose was to supplement traditional approaches to mathematics teaching and learning currently used by teachers in the schools (Maree et al. 2005). Teacher proficiency levels, which are a factor related to the

content as well as the application of pedagogical knowledge that needs to be mastered by mathematics teachers for teaching mathematics in schools, were investigated by this study through the sharing of ideas (Darling-Hammond & Sykes 2003; Darling-Hammond et al. 2017; Johnson & Kritsonis 2006).

Results discussed here form part of a larger qualitative study that investigated difficulties experienced by mathematics teachers in teaching mathematical concepts in schools. The study revealed that peer collaboration in early childhood mathematics teaching is key to helping teachers identify threshold concepts in mathematics that they had initially found difficult as individual teachers (See Figures 1–3). Collaboration helped them to teach the subject effectively at the FP level. This finding is in line with the claims by Jaworsky (1994), who explained that in teaching mathematical concepts, teachers' knowledge for presenting concepts should reference the importance of shaping learners' experiences, because this is where the impact of the human culture of understanding and acceptance occurs and where the rules and conventions of language use are constructed by an individual with the extensive functional outcomes manifested where learning occurs. This supports teachers' collaborative efforts when they share ideas with one another, as they are given the opportunity to address learners' problems effectively and identify mathematical concepts in teaching and learning. In the same way, Vygotsky's views on this theory contribute significantly to social constructivist epistemology, which dwells much on how learning is mediated in a collaborative manner in accordance with the context, and by sharing personal experiences with peers through open discussion in order to overcome teaching and learning obstacles. This type of learning is similar to social interaction, which concentrates on cognitive development of individuals through discussion and sharing of information (Lantolf & Thorne 2006; Lin 2015).

The study, furthermore, established that collaboration by mathematics teachers of different calibres is necessary to overcome the paucity of global mathematics teaching skills for childhood-level mathematics, in order to foster learners' knowledge of mathematical concepts and to stimulate their interest in the subject.

Sarason (1993) maintains that if one wants to change the sphere of learners, one needs to first change the education of teachers; hence teachers were fully involved in this research for almost 3 consecutive weeks, after which the intervention yielded fruitful results. Sarason (1993) maintains further that it is necessary to prepare educators for what life is like in classrooms, schools, school systems and society. It is an interesting phenomenon to embark on mathematics teachers' development through collaboration, which serves as a platform characterised by notions of negotiation and identification of certain topics that seem crucial for individual teachers who normally prepare their lessons in isolation. The purpose was to create a learner-centred approach to effective teaching and learning of mathematics, as advocated by Lewis (2016). Ni Shuilleabhain and Seery (2018) and Fullan and Hargreaves (1992) argue that in order for any relevant and successful fundamental change to occur in classrooms, teachers must be encouraged at all times to collaborate with their peers. Engaging teachers in such collaboration is therefore key to helping each other identify the key concepts in mathematics teaching.

It is believed that mathematics is a sequential process or development, fixed to a certain person, topic, environment or idea that changes or influences the life of that person through thinking and doing. Researchers such as Chamoso, Cáceres and Azcárate (2012) and Schon (1983) support this notion. Teachers came together and shared ideas in order to overcome the challenges posed by dealing with mathematical concepts. Within the boundaries of a PCK demonstration, Shulman (1986) explains that:

> [*T*]he knowledge for the most regularly taught topics in one's subject area, the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstration in a word, the ways of representing and formulating the subject make it comprehensible to others.

This indicates the vital importance of what the teacher knows, how much he or she knows of the content and that he or she knows how to present it so that learners understand. Peer discussion, thinking, doing and sharing of ideas should always go hand in hand in mathematics learning, as it is these activities that help stimulate learners' creativity in divergent ways through coaching (Vygotsky 1978). This could be even more effective when teachers meet as peers and discuss their experiences based on what they know and how they do it, in order to develop important concepts for teaching and learning of mathematics. There are various opinions on what it means for a teacher to know the content that must be taught or the appropriate way to present it to the learners for conceptual understanding (Pasley 2011). Research has demonstrated that teachers can develop themselves better professionally in teaching for mathematics by strengthening the relationship between their instructional practices and their underlying knowledge base through collaboration (Gellert 2008). Thus, communities of mathematics teachers at primary schools could improve their mathematics knowledge and routines of teaching the subject to help learners understand better if they engage in solid collaboration amongst themselves. Doing so will help teachers acquire new skills and knowledge, and they will be able to put their new visions of mathematically rich classroom activities into practice, where development of their knowledge base precedes the development of their instructional practices through the sharing of ideas (Gellert 2008; Gellett 2003).

Conclusions and recommendations

From the findings of this study, it can be concluded that teachers demonstrated different kinds of mathematical knowledge, knowledge of instruction and knowledge of curriculum to identify threshold concepts in mathematics. Through extensive collaboration, teachers can develop and acquire knowledge and skills relevant to tracking unnecessary misconceptions amongst learners in the mathematics classroom and hence develop an interest in

understanding mathematical concepts in everyday life. The study concluded that collaboration was beneficial for teachers in the following ways: it helps in providing coaching and expert support for teachers, which involves the sharing of expertise about content and evidence-based practices, and it focuses directly on teachers' individual needs, as well as their learners, for PD through content-focused discussion. It further incorporates active learning amongst teachers whereby they share their problems and find solutions through collaborative support. They make use of models of effective practices that offer sustainable feedback and reflection, which provide teachers with adequate time to learn, practise, implement and reflect upon new strategies to facilitate changes in their teaching practice. Based on the results, it is recommended that a teacher collaboration network should be organised for teachers. Teachers demand PD programmes such as workshops and in-service training to be fully implemented to assist the teachers to grow and develop professionally. Teachers also indicated that there is a need to enforce team-teaching amongst mathematics teachers, which encourages monitoring of the progress of all the mathematics teachers in the schools in the province. It is further recommended that collaborative class observation, discussion and mutual result reflection should be engaged in on a regular basis.

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Author's contributions

I declare that I am the sole author of this research article.

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